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Teleportation of A Single Qubit State via Unique W State

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We propose a scheme for teleporting a single qubit state employing a unique three-particle W state as quantum channel. By adopting QED cavity technologies, our scheme does not involve the Bell-state measurements(BMs). An unknown state $a|0\rangle + b|1\rangle$ can be probabilistically teleported by communicators' single particle measurements, unitary operations and classical communications. We can perfectly teleport quantum state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ with 100% probability.

KEY WORDS: teleportation; QED cavity; unique W state.

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1. INTRODUCTION

Quantum teleportation is a process to transmit an unknown state to a remote location via a quantum channel aided by some classical communications. Bennett *et al.* (1993) originally proposed the theoretical protocol for teleporting a single qubit using an entangled pair of spin-$\frac{1}{2}$ particles in 1993. Subsequently, large number of protocols of quantum teleportation are proposed with some pure entangled states or maximal multipartite entangled states (Ge and Shen, 2005; Sun *et al.*, 2005; Marinatto and Weber, 2000; Yang and Guo, 2000; Yan *et al.*, 2002). Experimental demonstrations of quantum teleportation have been realized by some groups (Bouwmeester *et al.*, 1997; Boschi *et al.*, 1998; Furusawa *et al.*, 1998; Kim *et al.*, 2001). Now people pay special interest to the physical realization of quantum teleportation and the key steps of which are generation of quantum channel and realization of the BMs.

In QED cavity, atoms interact with a quantized electromagnetic field. It has already been proven that QED cavity is a useful tool for testing fundamental quantum properties. Many schemes have been proposed for atomic state teleportation

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in cavity QED (Davidovich *et al.*, 1994; Jin *et al.*, 2005; Yang *et al.*, 2004; Yang and Cao, 2004; Zheng, 2004). Yang *et al.* (2004) have made a many-party controlled teleportation scheme using Bell states and multiqubit GHZ states as quantum channel, but it is difficult to generate multiqubit GHZ states in experiments. Zheng (2004) has presented a scheme for the approximate conditional teleportation of a single atomic state via a resonant atom-cavity interaction, but its probability of success is 0.25. Jin *et al.* (2005) have suggested a scheme of teleporting an unknown two-atom entangled state using W state as quantum channel in a driven cavity QED, the probability of success being 1.0.

Different kinds of quantum channels have been employed in quantum teleportation scheme, such as the Einstein-Podolsky-Rosen (EPR) pair (Bennett *et al.*, 1993; Yang *et al.*, 2004), the GHZ state (Sun *et al.*, 2005), the W state (Solano *et al.*, 2003; Jin *et al.*, 2005) and the squeezed vacuum state (Milburn and Braunstein, 1999). Bipartite entanglement has been extensively studied, while multipartite entanglement is still under extensive exploration. The entanglement of the W state is maximally robust under disposal of any one of the three qubits (Dür *et al.*, 2000). W state leads to a stronger non-classicality than GHZ state (SenDe *et al.*, 2003). It is convenient for experimental implementations that we directly use the W state and do not need to distill GHZ state from W state. In this paper, we propose a teleportation scheme for a single particle state via a unique three-particle entangled W state. The distinct advantages of the scheme are: Firstly, BMs are not required in the scheme (adopting QED cavity technologies); Secondly, the teleportation scheme is insensitive to both cavity decay and thermal field (introducing a strong classical driving field); Thirdly, the successful probability of teleporting state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is 1.0; Lastly, the teleportation scheme is feasible using today technologies. Two agents can successfully realize the teleportation scheme by operating single particle measurements, unitary operations and classical communications.

2. QED CAVITY MODEL

The QED cavity model used in our scheme describes two identical two-level atoms interacting simultaneously with a single-mode cavity field. During the interaction between atoms and the cavity field, a strong classical field is simultaneously accompanied. Consider the cavity field is initially in the vacuum state. In the rotating-wave approximation, the Hamiltonian (assuming $\hbar = 1$) for such a system is given by Zheng (2002); and Solano *et al.* (2003)

$$H = \omega_0 \sum_{j=1,2} S_{z,j} + \omega_a a^\dagger a + \sum_{j=1,2} [g(a^\dagger S_j^- + a S_j^+) + \Omega(S_j^+ e^{-i\omega_d t} + S_j^- e^{i\omega_d t})], \tag{1}$$

where $S_j^+ = |1\rangle_j\langle 0|$, $S_j^- = |0\rangle_j\langle 1|$ and $S_{z,j} = \frac{1}{2}(|1\rangle_j\langle 1| - |0\rangle_j\langle 0|)$ are atomic operators, $|1\rangle_j$ and $|0\rangle_j$ being the excited and ground states of the jth atom, respectively. ω_0 is atomic transition frequency, ω_a is cavity frequency and ω_d is the frequency of classical driving field. $a^\dagger$ (a) denotes the creation (annihilation) operator for the cavity mode. g is the atom-cavity coupling strength, and Ω is the Rabi frequency of the classical field. Assuming $\omega_0 = \omega_d$, the Hamiltonian, in the interaction picture, is

$$H_I = \sum_{j=1,2} [\Omega(S_j^+ + S_j^-) + g(e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+)], \tag{2}$$

where δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω_a.

In the interaction picture, the evolution operator of the system (Sen(De) *et al.*, 2003) is

$$U(t) = e^{-iH_0 t} e^{-iH_{\text{eff}} t}, \tag{3}$$

where $H_0 = \sum_{j=1,2} \Omega(S_j^+ + S_j^-)$, H_{eff} is the effective Hamiltonian. In the large detuning $\delta \gg \frac{g}{2}$ and strong driving field $2\Omega \gg \delta, g$ limit, there is no energy exchange between atoms and cavity and it is insensitive to both cavity decay and thermal field. The effective Hamiltonian, in the rotating-wave approximation, can be described as follows (Zheng, 2003):

$$H_{\text{eff}} = \lambda \left[\frac{1}{2} \sum_{j=1,2} (|1\rangle_j\langle 1| + |0\rangle_j\langle 0|) + \sum_{j,l=1,2,j\neq l} (S_j^+ S_l^+ + S_j^+ S_l^- + H.c.) \right], \tag{4}$$

where $\lambda = \frac{g^2}{2\delta}$.

Suppose atoms 1 and 2 are initially in one of four product states $|0\rangle_1|0\rangle_2$, $|0\rangle_1|1\rangle_2$, $|1\rangle_1|0\rangle_2$ and $|1\rangle_1|1\rangle_2$. When two atoms are sent into the single-mode driven QED cavity, the four states will undergo the following evolutions, respectively

$$\begin{aligned}|0\rangle_1|0\rangle_2 \rightarrow e^{-i\lambda t}[&\cos\lambda t(\cos\Omega t|0\rangle_1 - i\sin\Omega t|1\rangle_1)(\cos\Omega t|0\rangle_2 - i\sin\Omega t|1\rangle_2)\\ &-i\sin\lambda t(\cos\Omega t|1\rangle_1 - i\sin\Omega t|0\rangle_1)(\cos\Omega t|1\rangle_2 - i\sin\Omega t|0\rangle_2)],\end{aligned} \tag{5}$$

$$\begin{aligned}|0\rangle_1|1\rangle_2 \rightarrow e^{-i\lambda t}[&\cos\lambda t(\cos\Omega t|0\rangle_1 - i\sin\Omega t|1\rangle_1)(\cos\Omega t|1\rangle_2 - i\sin\Omega t|0\rangle_2)\\ &-i\sin\lambda t(\cos\Omega t|1\rangle_1 - i\sin\Omega t|0\rangle_1)(\cos\Omega t|0\rangle_2 - i\sin\Omega t|1\rangle_2)],\end{aligned} \tag{6}$$

$$\begin{aligned}|1\rangle_1|0\rangle_2 \rightarrow e^{-i\lambda t}[&\cos\lambda t(\cos\Omega t|1\rangle_1 - i\sin\Omega t|0\rangle_1)(\cos\Omega t|0\rangle_2 - i\sin\Omega t|1\rangle_2)\\ &-i\sin\lambda t(\cos\Omega t|0\rangle_1 - i\sin\Omega t|1\rangle_1)(\cos\Omega t|1\rangle_2 - i\sin\Omega t|0\rangle_2)],\end{aligned} \tag{7}$$

$$\begin{aligned}|1\rangle_1|1\rangle_2 \rightarrow e^{-i\lambda t}[&\cos\lambda t(\cos\Omega t|1\rangle_1 - i\sin\Omega t|0\rangle_1)(\cos\Omega t|1\rangle_2 - i\sin\Omega t|0\rangle_2)\\ &-i\sin\lambda t(\cos\Omega t|0\rangle_1 - i\sin\Omega t|1\rangle_1)(\cos\Omega t|0\rangle_2 - i\sin\Omega t|1\rangle_2)].\end{aligned} \tag{8}$$

We choose interaction time in such a way that $\lambda t = \frac{\pi}{4}$ and $\Omega t = \pi$ are fulfilled, then the joint Bell states will evolve, according to Eqs. (5–8), in the following way,

$$\begin{aligned}
|\phi^{+}\rangle_{12} &= \frac{1}{\sqrt{2}}(|00\rangle_{12} + i|11\rangle_{12}) \to |00\rangle_{12},\\
|\phi^{-}\rangle_{12} &= \frac{1}{\sqrt{2}}(|00\rangle_{12} - i|11\rangle_{12}) \to -i|11\rangle_{12},\\
|\psi^{+}\rangle_{12} &= \frac{1}{\sqrt{2}}(|01\rangle_{12} + i|10\rangle_{12}) \to |01\rangle_{12},\\
|\phi^{-}\rangle_{12} &= \frac{1}{\sqrt{2}}(|01\rangle_{12} - i|10\rangle_{12}) \to -i|10\rangle_{12}
\end{aligned} \tag{9}$$

where we have discarded the common phase factor $e^{-i\frac{\pi}{4}}$.

Hence, the BMs, which are complex for experimental realization, can be exactly converted into two separate atomic measurements.

3. TELEPORTATION OF A SINGLE PARTICLE STATE

A two-level atom 1 to be teleported is assumed to be the following superposition of the excited state $|1\rangle_1$ and the ground state $|0\rangle_1$,

$$|\phi\rangle_1 = a|0\rangle_1 + b|1\rangle_1. \tag{10}$$

To achieve this goal of quantum teleportation, we construct a unique three-atom entangled W state as quantum channel between sender Alice and receiver Bob. It is in the following form (Dür *et al.*, 2000)

$$|\Phi\rangle_{234} = \frac{1}{2}(|100\rangle_{234} + |010\rangle_{234} + |001\rangle_{234} + |000\rangle_{234}). \tag{11}$$

Alice holds atoms 1 and 2, and Bob owns atoms 3 and 4.

The state of the whole system involving atoms 1, 2, 3 and 4 can be written as

$$\begin{aligned}
|\Psi\rangle_{1234} &= |\phi\rangle_1 \otimes |\Phi\rangle_{234}\\
&= \frac{1}{2}(a|0\rangle_1 + b|1\rangle_1)(|100\rangle_{234} + |010\rangle_{234} + |001\rangle_{234} + |000\rangle_{234})\\
&= \frac{1}{2}(a|0100\rangle_{1234} + a|0010\rangle_{1234} + a|0001\rangle_{1234} + a|0000\rangle_{1234}\\
&\quad + b|1100\rangle_{1234} + b|1010\rangle_{1234} + b|1001\rangle_{1234} + b|1000\rangle_{1234})\\
&= \frac{1}{2\sqrt{2}}[(|\psi^{+}\rangle_{12} + |\psi^{-}\rangle_{12})a|00\rangle_{34} + (|\phi^{+}\rangle_{12} + |\phi^{-}\rangle_{12})(a|10\rangle_{34}\\
&\quad + a|01\rangle_{34} + a|00\rangle_{34}) + (|\phi^{+}\rangle_{12} - |\phi^{-}\rangle_{12})b|00\rangle + (|\psi^{+}\rangle_{12}\\
&\quad - |\psi^{-}\rangle_{12})(b|10\rangle_{34} + b|01\rangle_{34} + b|00\rangle_{34})]
\end{aligned}$$

$$= \frac{1}{2\sqrt{2}}[|\psi^+\rangle_{12}(a|00\rangle_{34} + b|10\rangle_{34} + b|01\rangle_{34} + b|00\rangle_{34}) + |\psi^-\rangle_{12}(a|00\rangle_{34} - b|10\rangle_{34} - b|01\rangle_{34} - b|00\rangle_{34}) + |\phi^+\rangle_{12}(a|10\rangle_{34} + a|01\rangle_{34} + a|00\rangle_{34} + b|00\rangle_{34}) + |\phi^-\rangle_{12}(a|10\rangle_{34} + a|01\rangle_{34} + a|00\rangle_{34} - b|00\rangle_{34})] \tag{12}$$

where $|\phi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle_{12} \pm |11\rangle_{12})$, $|\psi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}(|01\rangle_{12} \pm |10\rangle_{12})$ are Bell states of atoms 1 and 2.

Firstly, Alice sends simultaneously atoms 1 and 2 to the QED cavity driven by a strong classical field. Adjusting the interaction time ($\lambda t = \frac{\pi}{4}$) and modulating the driving field ($\Omega t = \pi$), according to Eq. (9), the state of the system in Eq. (12) will evolve into:

$$|\Psi\rangle_{1234} = \frac{1}{2\sqrt{2}}\{|01\rangle_{12}[(a|0\rangle_3 + b|1\rangle_3)|0\rangle_4 + b|01\rangle_{34} + b|00\rangle_{34}] + |10\rangle_{12}[(a|0\rangle_3 - b|1\rangle_3)|0\rangle_4 - b|01\rangle_{34} - b|00\rangle_{34}] + |00\rangle_{12}[(a|1\rangle_3 + b|0\rangle_3)|0\rangle_4 + a|01\rangle_{34} + a|00\rangle_{34}] + |11\rangle_{12}[(a|1\rangle_3 - b|0\rangle_3)|0\rangle_4 + a|01\rangle_{34} + a|00\rangle_{34}]\} \tag{13}$$

Then Alice detects atoms 1 and 2 separately. If Alice's detecting outcome is $|01\rangle_{12}$, atoms 3 and 4 will collapse into the state $(a|0\rangle_3 + b|1\rangle_3)|0\rangle_4 + b|01\rangle_{34} + b|00\rangle_{34}$. Through a classical channel Alice communicates the outcome of her measurement to Bob, then Bob measures atoms 4 on basis $\{|0\rangle, |1\rangle\}$. The total successful probability of this scheme is

$$P = \left(\frac{1}{2\sqrt{2}}\right)^2 \times 2 \times \frac{1}{1+2b^2} + \left(\frac{1}{2\sqrt{2}}\right)^2 \times 2 \times \frac{1}{1+2a^2} = \frac{1}{4}\left(\frac{1}{1+2b^2} + \frac{1}{1+2a^2}\right). \tag{14}$$

When Alice's outcome is $|10\rangle_{12}$, $|00\rangle_{12}$ or $|11\rangle_{12}$, Bob must operate unitary transformation σ_z, σ_x and $\sigma_z \otimes \sigma_x$, respectively, on collapsed state $a|0\rangle_3 - b|1\rangle_3$, $a|1\rangle_3 + b|0\rangle_3$ and $a|1\rangle_3 - b|0\rangle_3$ to replicate the teleported unknown state.

When $a = b = \frac{1}{\sqrt{2}}$, that is $|\phi\rangle_1 = \frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1)$, $|\Psi\rangle_{1234}$ becomes

$$|\Psi\rangle_{1234} = \frac{1}{2\sqrt{2}}(|0\rangle_1 + |1\rangle_1)(|100\rangle_{234} + |010\rangle_{234} + |001\rangle_{234} + |000\rangle_{234}) = \frac{1}{4}[|\psi^+\rangle_{12}(2|00\rangle_{34} + |01\rangle_{34} + |10\rangle_{34}) - |\psi^-\rangle_{12}(|01\rangle_{34} + |10\rangle_{34}) + |\phi^+\rangle_{12}(2|00\rangle_{34} + |01\rangle_{34} + |10\rangle_{34}) + |\phi^-\rangle_{12}(|01\rangle_{34} + |10\rangle_{34})] \tag{15}$$

When the state of atoms 3 and 4 collapses into the state $2|00\rangle_{34} + |01\rangle_{34} + |10\rangle_{34}$, which can be rewritten as $\sqrt{2}(|0\rangle_3|x_+\rangle_4 + |x_+\rangle_3|0\rangle_4)$, where $|x_\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, Bob can replicates the teleported state by measuring any one of atoms 3 and 4 on basis $\{|x_+\rangle, |x_-\rangle\}$. On the other hand, when atoms 3 and 4 is projected into the state $|01\rangle_{34} + |10\rangle_{34}$, Bob needs to make a two-qubit C-not transformation(C_{NOT}) on atoms 3 and 4. C_{NOT} can be described as in matrix

$$C_{\mathrm{NOT}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{16}$$

and

$$C_{\mathrm{NOT}}|01\rangle_{34} = |01\rangle_{34}, C_{\mathrm{NOT}}|10\rangle_{34} = |11\rangle_{34}. \tag{17}$$

Then $C_{\mathrm{NOT}}(|01\rangle_{34} + |10\rangle_{34}) = |01\rangle_{34} + |11\rangle_{34} = \sqrt{2}|x_+\rangle_3|1\rangle_4$. Bob can construct the initial state of atom 1 by measuring atom 3 on basis $\{|x_+\rangle, |x_-\rangle\}$ after he receives Alice's classical information. The total probability of successful teleportation is 1.0.

4. CONCLUSIONS

In conclusion, we have proposed a theoretical scheme for the teleportation of a single particle quantum state via QED cavity, in which a unique three-particle entangled W state serves as quantum channel. Due to the atoms in the single-mode cavity field are driven by a strong classical field, the teleportation scheme is insensitive to both cavity decay and thermal field. The scheme does not require the BMs on two atoms and can be probabilistically realized by communicators' single particle measurements, unitary transformations and classical communications when the the teleported state is $a|0\rangle + b|1\rangle$. Meanwhile, when $a = b = \frac{1}{\sqrt{2}}$, the probability of successful teleportation is 1.0.

This proposed scheme is feasible using today's technologies. Eibl *et al.* (2004) has experimentally realized a three-qubit entangled W state. The required unique three-qubit entangled W state can be obtained by local logic gates. In addition, in our scheme we must distinguish two atoms after they fly out of the QED cavity. Riebe *et al.* (2004) have proposed a technique to address any specified target ion using tightly focused laser beams and to 'hide' the remaining ions from the target ionfls fluorescence by changing their internal states so that they are insensitive to the fluorescent light.

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